3.5 Applications of Extrema (Minimum Homework: 1, 3, 5)

1) A company makes a single product. The cost function for the product is given by:
$C(x) = 0.5x^2 + 50x + 200$ where $C(x)$ is the total cost to produce x units of the product.
The demand function is given by $p(x) = -x + 110$, where $p(x)$ is the price to sell x units of the product.

1a) Create a revenue function.
1b) Create a profit function.
1c) How many units must the company produce and sell to maximize profit?
1d) What is the maximum profit?
1e) What price per unit must be charged to make maximum profit?

a) $R(x) = (-1x+110)x$

$R(x) = -1x^2+110x$

b) $P(x) = -1x^2+110x-(0.5x^2+50x+200)$

$P(x) = -1x+110x-0.5x^2-50x-200$

$P(x) = -1.5x^2+60x-200$

c) $P'(x) = -3x+60$ 20 units

$-3x+60=0$

$-3x=-60$

$x=20$

d) $P(20) = -1.5(20)^2+60(20)-200 = \$400$

e) $p(20) = -1(20)+110 = -20+110 = \$90$

1a) Create a revenue function. $R(x) = -1x^2 + 110x$
1b) Create a profit function. $P(x) = -1.5x^2 + 60x - 200$
1c) How many units must the company produce and sell to maximize profit? 20 *units*
1d) What is the maximum profit? $400
1e) What price per unit must be charged to make maximum profit? $90

3) The marketing research department of Shank, a quarterly magazine for beginning golfers, has determined that the price-demand equation for the magazine is approximated by

$p(x) = -0.1x + 200$

where x represents the number of magazines printed and sold each quarter, in hundreds, and $p(x)$ is the price, in dollars, of the magazine. The cost of printing, distributing, and advertising is given by

$C(x) = 0.2x^2 + 50x + 3000$

3a) Create a revenue function.
3b) Create a profit function.
3c) How many units must the company produce and sell to maximize profit?
3d) What is the maximum profit?
3e) What price per unit must be charged to make maximum profit?

a) $R(x) = (-0.1x + 200)(x)$

$R(x) = -0.1x^2 + 200x$

b) $P(x) = -0.1x^2 + 200x - (0.2x^2 + 50x + 3000)$

$-0.1x^2 + 200x - 0.2x^2 - 50x - 3000$

$\cancel{P(x) = -0.1x^2}$

$P(x) = -0.3x^2 + 150x - 3000$

c) $P'(x) = -0.6x + 150$

$-0.6x + 150 = 0$

$\dfrac{-0.6x}{-0.6} = \dfrac{-150}{-0.6}$

$\boxed{x = 250 \text{ units}}$

3a) Create a revenue function. $R(x) = -0.1x^2 + 200x$
3b) Create a profit function. $P(x) = -0.3x^2 + 150x - 3000$
3c) How many units must the company produce and sell to maximize profit? $250\ units$
3d) What is the maximum profit? $15,750
3e) What price per unit must be charged to make maximum profit? $175

d) $P(250) = -0.3(250)^2 + 150(250) - 3000$

$\$15,750$

e) $p(250) = -0.1(250) + 200 = \boxed{\$175}$

5) The daily production cost $C(x)$ for a factory to manufacture x deluxe contour chairs is given to be $C(x) = \frac{1}{2}x^2 + 14x + 500$. The price demand function is $p(x) = -\frac{3}{2}x + 150$ where $p(x)$ is the price needed to sell x − chairs.

a) Create a revenue function.
b) Create a profit function.
c) How many units must the company produce and sell to maximize profit?
d) What is the maximum profit?
e) What price per unit must be charged to make maximum profit?

(a) $R(x) = \left(-\frac{3}{2}x + 150\right)x$

$$\boxed{R(x) = -\frac{3}{2}x^2 + 150x}$$

(b) $P(x) = -\frac{3}{2}x^2 + 150x - \left(\frac{1}{2}x^2 + 14x + 500\right)$

$P(x) = -\frac{3}{2}x^2 + 150x - \frac{1}{2}x^2 - 14x - 500$

$$\boxed{P(x) = -2x^2 + 136x - 500}$$

c) $P'(x) = -4x + 136$

$-4x + 136 = 0$

$\frac{-4x}{-4} = \frac{-136}{-4}$

$x = 34$

$\boxed{\text{34 units}}$

d) $P(34)$

$= -2(34)^2 + 136(34) - 500 = \boxed{1812}$

5a) Create a revenue function. $R(x) = -\frac{3}{2}x^2 + 150x$
5b) Create a profit function. [redacted] $P(x) = -2x^2 + 136x - 500$
5c) How many units must the company produce and sell to maximize profit? [redacted] 34 units
5d) What is the maximum profit? [redacted] \$1812
5e) What price per unit must be charged to make maximum profit? [redacted] \$99

$p(34) = -\frac{3}{2}(34) + 150 = \boxed{\$99}$